

Mathematics of Forensic DNA Identification

World Trade Center Project

Extracting Information from Kinships and Limited Profiles

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Introduction

- 2,795 people were killed in the World Trade Center attacks on September 11, 2001.
- 20,000 remains were recovered, the vast majority of which would require DNA matching for identification.
- Existing software tools for DNA identification proved wholly inadequate for the scope and magnitude of this project.

Timeline

- **September 17:** Armed Forces DNA Identification Lab [AFDIL] asks Gene Codes to update *Sequencher*[™] for the Pentagon and Shanksville crashes.
- **September 28:** Office of the Chief Medical Examiner [OCME] in New York City contacts us for new software.
- **October 15:** Using the *Extreme Programming* [XP] methodology, software development is underway.
- **December 13:** *M-FISys* (Mass-Fatality Identification System) has its first release to the OCME.
- **Since:** Weekly releases personally delivered to the OCME, to accommodate rapidly changing requirements.

Identification Technologies

- Technologies used for Identification
 - STR
 - mtDNA
 - SNP
- Methods used:
 - Direct Match to a Personal Effect
 - Kinship Analysis

STR: Short Tandem Repeats

- A repeat of a short sequence of bases (4 or 5)
- For example, at locus position D7S280, it is the four base sequence gata we look for:
...gatagatagatagatagatagatagatgtttatctc...
- In the above example, gata is repeated 6 times with a 3-base partial repeat.
- “6.3” is therefore assigned for this allele.
- Being diploid, we have two alleles per locus, thus (up to) two values are stored, e.g. 6.3/8.

STR Frequency

- In 1997, the FBI standardized on 13 STR loci used in the national database, *CoDIS*.
- Frequency data for each locus/allele value is available for various races. For example:

Locus:	D16S539	TPOX	D3S1358	FGA	D7S820	vWA	D13S317	TH01
Allele:	11/13	8	15.2	21/13.2	10/11	15.2	11/12	9.3
Freq:	8.55%	39.4%	0.099%	0.796%	14.6%	0.099%	18.2%	9.21%

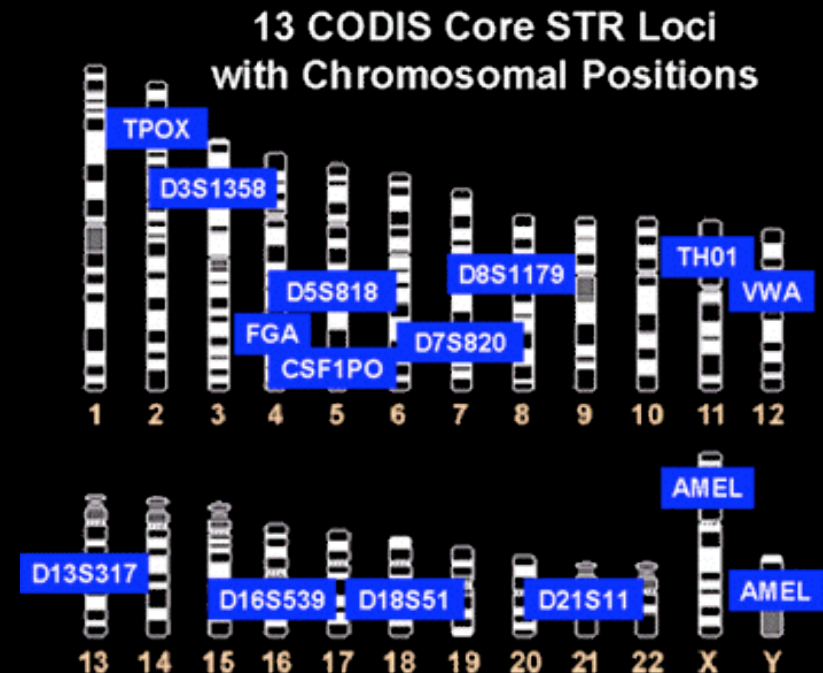
- Since STR loci are independent, these frequencies can be multiplied: 5.6×10^{-13}
- Likelihood = $1 / \text{Frequency} = 1.8 \times 10^{12}$

STR Profiles

- *M-FISys* STR profile contains 16 elements:

- Amelogenin (Gender)
- 13 CoDIS Core Loci
- 2 PowerPlex Loci:
 - Penta D
 - Penta E

- Minimum Likelihood:
 7.6×10^{15}



STR Likelihood Threshold

- OCME wants a minimum likelihood for identification which ensures a chance of a mismatch to be less than 1 in a million.
- Assuming a population of 5000, what is the smallest n such that a 10^n min likelihood yields a mismatch prob $< 10^{-6}$?
- Since likelihood is the inverse of probability, $p = 1 / 10^n$
- The probability of no mismatch is $q = 1 - p = 1 - 1 / 10^n$
- The prob. of no mismatch in 5000 = $1 - q^{5000} = 1 - (1 - 1/10^n)^{5000}$
- Thus we have the inequality:
$$1 - (1 - 1 / 10^n)^{5000} < 1 / 1,000,000$$
- Solving for n we get:
$$n > -\log_{10}(1 - (1 - 10^{-6})^{1/5000}) = 9.699$$
- Therefore we set $n = 10$.

Direct STR Identification

- A victim remain (called a disaster sample) can be identified by direct match if its profile is either:
 - complete and matches Personal Effects (2 modalities)
 - partial with no mismatches, with a likelihood $\geq 10^{10}$ amongst common loci
- A sample was further investigated if its STR profile likelihood $\geq 10^{10}$ and with either:
 - a single mismatch only, supported by Kinship
 - mismatches due only to allelic dropout

Partial Profiles

- All STR profiles containing at least 11 CoDIS markers or more will have likelihoods $\geq 10^{10}$
- 70% of the victim samples yielded partial profiles (missing at least one CoDIS marker)
- 25% of these partial profiles had likelihood values $\geq 10^{10}$
- Leaving half of victim samples which cannot be identified through STR means alone (using these parameters).

STR Likelihood: Locus Probability

- Likelihood = $1 / \text{Probability Frequency}$
- OCME has locus-allele frequency data
- Locus Probability can be first approximated by ignoring population structure and using the *Hardy-Weinberg proportions*:
 - p^2 for homozygous alleles: p = frequency of allele
 - $2pq$ for heterozygous alleles: p, q = frequency of each allele
- Above assumes an infinite population with random mating

STR Likelihood: θ

- Because the population is finite, we introduce the inbreeding coefficient θ
- Factoring this into the H-W equations:
 - $p^2 + p(1-p)\theta$ for homozygous alleles
 - $2pq(1-\theta)$ for heterozygous alleles
- Because θ is very small, $1-\theta$ is close to 1, we round it to remain conservative:
 - $p^2 + p(1-p)\theta$ for homozygous alleles
 - $2pq$ for heterozygous alleles
- OCME chooses the standard $\theta = 0.03$

STR Likelihood: Profiles

- Once we have calculated the probability frequency for each locus, we can calculate the likelihood of the entire profile:
- If $P_k (A_k)$ is the probability of allele A at locus k , we can define the likelihood of STR profile S as:

$$L(S) = \prod_{k \in \text{Alleles}} 1 / P_k (A_k)$$

- Note that this works even for partial profiles

STR Likelihood: Race

- OCME has frequency values for four population groups: Asian, Black, Caucasian & Hispanic
- Cannot always rely on reported race, and the race is unknown for a disaster sample
- *M-FISys* computes the Likelihood value across all four races and chooses the lowest value, just to be on the safer, more conservative side.

M-FISys STR Master List

M-FISys 4.0.2-Master List-Default

Locate: ID Samples: 12 Identified Aggregates: 1 Identifiable Aggregates: 1 Unidentified Aggregates: 2

ID	RM	Likelihood	I	# M Sn	Gen	D8S1328	vWA	FGA	D8S1179	D21S11	D18S51	D5S818	D13S317	D7S820	D16S530	TH01	TPOX	CSF1PO	Penta D	Penta
+ RM# 5913 (6)		2.8E+017	0		XY	14/17	16/17	23/25	13	30	13/14	11	8/11	11/13	7/9	8/11	11/13	neg	-	-
- RM# 4141 (4)		6.2E+020	1		XX	14/17	17	23/24	15/17	28/32	13/15	12	10/11	12	12/13	9	8/11	10/13	11/12	12/14
SP-90003-1	4141	6.2E+020		16	XX	14/17	17	23/24	15/17	28/32	13/15	12	10/11	12	12/13	9	8/11	10/13	11/12	12/14
DM0193281	4141	6.2E+020		16	XX	14/17	17	23/24	15/17	28/32	13/15	12	10/11	12	12/13	9	8/11	10/13	11/12	12/14
DM0196708	4141	6.2E+020		16	XX	14/17	17	23/24	15/17	28/32	13/15	12	10/11	12	12/13	9	8/11	10/13	11/12	12/14
- VIRT- DM0180017	4141	6.2E+020	1	16	XX	14/17	17	23/24	15/17	28/32	13/15	12	10/11	12	12/13	9	8/11	10/13	11/12	12/14
DM0180018				16	XX	14/17	17	23/24	15/17	28/32	13/15	12	10/11	12	12/13	9	8/11	10/13	11/12	12/14
OX01- DM0180018	4141	6.2E+020	1	16	XX	14/17	17	23/24	15/17	28/32	13/15	12	10/11	12	12/13	9	8/11	10/13	11/12	12/14
RM#4141: Simpson, Marge (Chain 1) Method: Test																				
- RM# 9112 (3)		1.6E+018	0		XX	13/16	16/17	20/22	13/15	28/29	12	12/13	9/10	11/12	8/11	6/9	10	10/12	-	-
SP-80007-1	9112	1.9E+019		12	XX	13/16	16	20/22	13/15	28/29	12	12/13	9/10	11/12	8	6/9	10	10/12	-	-
SP-80007-2	9112	1.9E+019		12	XX	13/16	16	20/22	13/15	28/29	12	12/13	9/10	11/12	8	6/9	10	10/12	-	-
DM0190020		1.6E+018		14	XX	13/16	16/17	20/22	13/15	28/29	12	12/13	9/10	11/12	8/11	6/9	10	10/12	-	-
- AS040004 (4)		4.9E+015	0		XY	16/17	17	20/21	10/11	30/31	15/16	10/12	8/11	11	11/13	7/9	8	8/11	-	-
DM0190066		4.9E+015		14	XY	16/17	17	20/21	10/11	30/31	15/16	10/12	8/11	11	11/13	7/9	8	8/11	-	-
DM0190221		4.9E+015		14	XY	16/17	17	20/21	10/11	30/31	15/16	10/12	8/11	11	11/13	7/9	8	8/11	-	-
DM0190881		4.9E+015		14	XY	16/17	17	20/21	10/11	30/31	15/16	10/12	8/11	11	11/13	7/9	8	8/11	-	-
DM0190889		4.9E+015		14	XY	16/17	17	20/21	10/11	30/31	15/16	10/12	8/11	11	11/13	7/9	8	8/11	-	-

Expand All Collapse All Hide Identical Alleles Exclude... Merge Export Hide Names Print Options...

STR mtDNA SNP Jobs

STR: Kinship Analysis

- Many times there was not sufficient data to perform an STR direct match.
- Cheek swabs from family members of missing persons are taken, and a pedigree tree in *M-FISys* can be generated.
- Likelihoods are calculated on victim samples to determine to which pedigree(s) they belong.
- Kinship Analysis was not performed if more than one relative was in the victim list.

Kinship Analysis: Likelihood

- As with direct STR, Kinship Likelihood is:
 - the product of Locus Likelihoods over common loci
 - the Likelihood Ratio $\geq 10^6$
 - calculated across all four races, using the lowest, most conservative value
 - uses frequency data from the OCME
- Analysis was performed for these relations:
Parent-Child, Full Sibling, Half Sibling

Kinship Algorithm

- *M-FISys* uses the Kinship algorithm as implemented by Dr. George Carmody of *Carleton University*
- Kinship Locus Likelihood defined as:

$$k = r_2x_2 + r_1x_1 + r_0x_0$$

- where the r_i 's are relationship proportions:

Parent-Child: $r_2 = 0$ $r_1 = 1$ $r_0 = 0$

Full Sibling: $r_2 = 1/4$ $r_1 = 1/2$ $r_0 = 1/4$

Half Sibling: $r_2 = 1/2$ $r_1 = 1/2$ $r_0 = 0$

First Cousin: $r_2 = 3/4$ $r_1 = 1/4$ $r_0 = 0$

Kinship Algorithm

- and with p & q the frequencies of the high & low alleles resp., the x_i 's are defined as:

$X_2 = p^2$	if victim is homozygous and matches an allele
$= 2pq$	otherwise
$X_1 = 0$	if relative & victim share no common allele
$= p$	if relative homozygous & shares low allele
$= q$	if relative homozygous & shares high allele
$= p/2$	if relative heterozygous & shares low allele
$= q/2$	if relative heterozygous & shares high allele
$= (p+q)/2$	if relative & victim are identical
$X_0 = 1$	if relative & victim alleles are identical
$= 0$	otherwise

M-FISys Kinship Form

M-FISys 4.0.2-Family Display-Default

Family: Profiles

#1

Victim

- BODE-DM0193979
- DM0190133
- BODE-DM0196054
- DM0192474
- DM0192245
- DM0192614
- VIRT-DM0191282

Pedigree

```

    graph TD
      F[F] --- M((M))
      F --- V((V))
      M --- V
      F --- A((A))
      M --- A
  
```

Reported

	BODE-DM0103979	BM-00609 #6	BF-00609 #2	BU-00609 #8
Gen	XX	XX	XY	XX
D3S1358	15	15/16	15	15
vWA	16/19	16/19	16/17	16/19
FGA	23/24	21/24	22/23	23/24
D8S1179	11/15	11/12	11/15	11/15
D21S11	31	31/32.2	31	31
D18S51	18	18	12/18	18
D5S818	11/13	11/13	11	11/13
D13S317	9/12	9	12/13	9/12
D7S820	7/8	7/10	8/10	7/8
D16S539	11/12	11/13	11/12	11/12
TH01	9/9.3	9.3	6/9	9/9.3
TPOX	9/10	9/10	9/11	9/10
CSF1PO	9/12	9/10	10/12	9/12
Penta D	neg	neg	neg	neg
Penta E	neg	neg	neg	neg
min LR to V	3.5E+019	5.2E+007	5.9E+003	5.5E+013

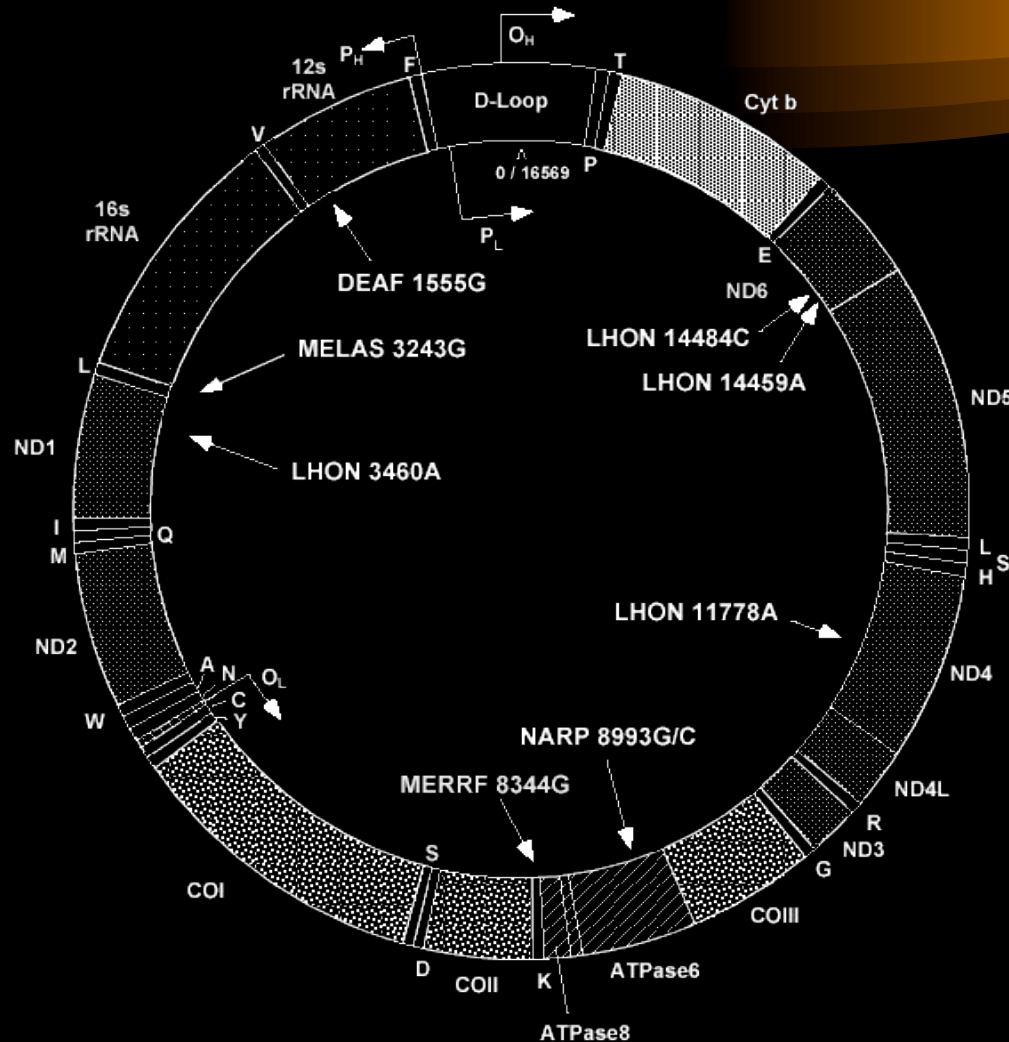
Identification Method: unidentified

Kinship Work List

Mitochondrial DNA Analysis

- Some victim samples were so degraded that sufficient STR data was not available for either direct STR match or Kinship analysis.
- mtDNA is hardier material, surviving under conditions which nuclear DNA degrades
- mtDNA is a 16,569-based circular genome.
- It is maternally inherited, and thus not unique.
- 5% of the Caucasian population share the same common mitotype.

mtDNA Map



mtDNA Analysis

- Mito-typing involves direct sequencing of two highly variable regions of mtDNA.
- The two areas used for mitotyping (HV1 & HV2) are not in a coding region.
- Only a sample's differences from *the Anderson Sequence* (an internationally accepted standard) need be tracked.
- However, 25% of the WTC victims had no maternally-related kin samples.

Mito Likelihood

- To determine likelihood for a given mitotype, we begin by counting its frequency x in the FBI mtCoDIS data of size n .
- The 95% confidence interval for a population proportion with Binomial distribution is estimated by the formula:
$$[\mu - 1.96\sigma/\sqrt{n}, \mu + 1.96\sigma/\sqrt{n}]$$
where μ is the mean and σ is the standard deviation.
- Since the probability p is just the number database hits, we set $p = x/n$, and so we have $\mu = p$ and $\sigma = \sqrt{p(1-p)}$.
- Thus we have as the upper bound: $x/n + \sqrt{x(n-x)/n}$.
- If there are no database entries, we use: $1 - \alpha^{1/n}$ with $\alpha = 0.05$
- Likelihood = 1 / Frequency

M-FISys mtDNA Form

M-FISys 4.0.2-Master List-Default Refresh Needed

Locate ID

ID	RM	Likelihood	I	#							
+					16111	16469	49	228	460	505	515
+					16111	16469	49	228	460	505	515
+					16111	16469	49	228	460	505	505.1
-	4				16111	16469	49	228	460	505.1	515
○ OMC1-DM010	2	1.0E+000		3	C	A	C	-	-	-	-
<div style="border: 1px solid black; padding: 2px;"> RM# 9123: Simpson, Homer (Chain 3) Method: Test Load date: 9/11/2001 12:00:00 AM </div>											
○ OMC1-DM0150005		3.0E+002	I		C	A	-	-	-	-	-
○ OMC1-DM0190026		1.7E+007		7	C	A	C	T	C	N	G
○ OMC1-DM0190027		7.0E+010		7	C	A	C	T	C	C	G
○ OMC1-DM0190029		2.0E+003		2	C	A	-	-	-	-	-
+	5				16111	16469	49	228	460	505	505.1

Expand All Collapse All Compare Hide Names Options...

STR mtDNA SNP Jobs

Introduction to SNP's

- Single Nucleotide Polymorphisms
- Represents single base differences
- Work pioneered by the GeneScreen division of Orchid Biosciences
- By being able to collect data from very short sequences, this technology offers a great deal of hope for the identification of badly degraded samples

SNP Selection

- SNP's occur on average every 100-300 bases within the human genome.
- 2 out of every 3 SNP's involve replacing a C with a T.
- Of these, there is a panel of 70 which are chosen, specifically those in which C and T are equally likely.

SNP Likelihood

- A complete profile of 70 SNP's each with an *independent* probability of 1/2 would yield a likelihood of match at $2^{70} \approx 10^{21}$.
- The probabilities are independent if the SNP's are *unlinked*, which we define to be at least 50MB apart.
- Unfortunately, it is **not** possible to have 70 SNP's 50MB apart in a 3GB genome.

SNP Independence

- A study by Dr. Ranajit Chakraborty of the *Center for Genome Information* concluded:
 - Allelic dependence is very low: 5.71% as compared to 5% expected by chance alone
 - Average heterozygosity of 46% across three population groups: Causian, Black, Hispanic
 - Despite lack of theoretically independent loci, his study supports the use of this 70 SNP panel for identification purposes

Non-Equiprobable SNP's

- Conservative likelihoods can be calculated even without the assumption of equi-probability.
- All bi-allelic heterozygous alleles have a minimum likelihood of 2, regardless of frequency:
$$f = 2pq = 2p(1-p) \leq 0.5 \quad \forall p \in [0,1]; \quad \therefore L = 1/f \geq 2$$
- The minimum likelihood of a SNP profile containing n heterozygous alleles is thus 2^n .
- As Forensic Mathematician Charles Brenner notes, even if the SNP frequencies were 0.1 and 0.9, 99% of cases will have 10 heterozygous loci out of 100.

M-FISys SNP Form

M-FISys 5.01-Master List-Default

Locate ID

ID	RM	Likelihood	I	#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35			
BD-55555 #5					TC	TC	TT	CC	TC	CC	CC	TC	TC	TC	TT	TT	TT	CC	TC	CC	CC	CC	TT	TC	TT	TC	CC	TC	TT	TC	TC	TC	CC	TT	TC	CC	TC	TT	TC	TC		
BF-55555 #5					CC	CC	TC	TC	TC	CC	TT	CC	TT	TC	TC	TC	TC	CC	CC	TC	CC	TC	TC	CC	TT	TT	TC	TT	TC	TC	TC	CC	TC	CC	TC	TC	TC	CC	TC	CC	TC	CC
BS-55555 #5					TC	TT	TC	TC	TC	CC	TC	CC	TT	TC	TC	TC	TT	TT	TT	TC	TT	CC	CC	TC	TC	TT	CC	TT	TC	TC	CC	TT	TC	TC	TC	TC	TC	TC	TC	TC	CC	TC
BU-55555 #5					TT	TC	TC	TT	TT	CC	TC	TT	CC	TC	TT	TT	TC	TT	CC	TC	TT	TC	TC	TT	TT	TT	CC	TC	TT	TT	TC	TC	TT	TT	TC	CC	TC	TT	TC	TC	TC	TC
PR-55555 #5					TC	TC	TC	CC	TC	CC	TC	TC	TC	TC	TC	TT	TT	CC	TC	TC	CC	CC	TT	TT	TT	TC	neg	TC	TC	TC	TT	CC	TC	TC	TC	CC	TC	CC	TT	CC	TC	TC

Expand All Collapse All Hide SNP colors Hide Names Options...

STR mtDNA **SNP** Jobs FB-2065

Combining Technologies for Partial Profiles

- The *M-FISys* software package is designed for rapid cross-pollination of STR, Kinship, mtDNA and SNP data of DNA samples.
- Consistent or conflicting data in one technology can help determine experimental errors resulting in another technology.
- *M-FISys* also generates Quality Control reports for finding such inconsistencies.

Combining SNP's & STR's

- By selectively choosing SNP's which are unlinked to each other and existing STR loci, independent likelihoods can be multiplied.
- With the exception of CSF1PO & D5S818, all STR loci are on different chromosomes.
- Thus any unlinked SNP's on an unused chromosome can be included in likelihood calculations.
- STR profiles below threshold are missing ≥ 3 loci
- Even if only 10 SNP's are used, the likelihood can be increased by 3 orders of magnitude! ($2^{10} \approx 10^{+3}$)

More Information

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<http://www.genecodes.com>

Updated Slides:

<http://www.jonhoyle.com/GeneCodes>

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Thank You!